PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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Ninety-Fourth Session.

Monday, 15th January 1877.

Sir WILLIAM THOMSON, President, in the Chair.

The following Communications were read:—

1. Communication from the President, relative to the Administration of the Government Fund of £4000.

2. On New and Little-known Fossil Fishes from the Edinburgh District. No. II. By R. H. Traquair, M.D., F.G.S.

Genus *Elonichthys*, Giebel, 1848.

Amblypterus (pars), Agassiz.
Palæoniscus (pars), Agassiz.
Pygopterus (pars), Agassiz.

Body fusiform, sometimes rather deep; median fins large, the caudal deeply cleft, inequilobate; dorsal and anal triangular, acuminate, the dorsal situated nearly opposite the interval between the ventrals and the anal; base of ventrals not extended; pectoral rays articulated (except a varying amount of the commencement of the principal ones). Fin rays ganoid, closely set, striated; fulcra closely set, minute. Scales sculptured, of moderate size, rhomboidal, their posterior borders frequently denticulated or serrated; the anterior overlapped area very narrow, reduced to a mere margin. Suspensorium very oblique; gape extensive; operculum well developed, oblong; interoperculum square-shaped; no suboperculum. Branchiostegal rays numerous; the anterior one of each lateral series broader than the rest a median lozenge-shaped plate in front. Jaws stout; teeth acutely conical, sharp, of two sizes, large and small, the larger ones being placed in a row internal to the more closely set outer series of smaller teeth. Ornament of cranial bones tubercular or rugose-tubercular; facial bones, and those of the shoulder-girdle striated; the jaws usually also tuberculated just at the dental margin.

The name *Elonichthys* was first proposed by Giebel* for certain fishes (*E. Germari*, *crassidens* and *lævis*) from the coal-measures of Wettin, near Halle, which he considered intermediate in generic character between *Amblypterus* and *Palæoniscus*, as defined by Agassiz. They were said to resemble *Palæoniscus* in their fulcrated fins, but to differ from it in the absence of the scaly covering to the rays, affirmed by Agassiz to exist in some Palæonisci, while to certain "*Amblypteri*" they showed an affinity in the striation of their scales, and to "*Amblypterus*" in general in the large size of the fins. Their special characteristics were to be found in the dentition, which consisted of an external series of minute teeth comparable to the "Bürstenzähne' of *Amblypterus*, between which there were large conical teeth, "wie ich dieselben weder bei den Palæonisken noch Amblypteren finde." Unfortunately, however, for this diagnosis, the fin-rays of *Palæoniscus* are no more covered with scales than those of any other genus belonging to this family; nor are the fins of Agassiz's "*Amblypteri*" destitute of fulcra "except on the upper lobe of the tail," as has been so repeatedly stated by compilers, who, copying this error from the "Tableau synoptique des Genres et des Espèces" have apparently overlooked the correction of it made by Agassiz himself a few pages further on in his general description of the genus.† The dentition, too, of Giebel's *Elonichthys* is essentially similar to that of Agassiz's *Amblyterus macropterus* (*Rhabdolepis* Troschel) in which large conical teeth were shown to exist by Goldfuss in 1847,‡ and again by

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* Fauna der Vorwelt, 1, 3, p. 249.

† Poissons Fossiles, vol. ii. pt. 1. p. 29.

‡ Beiträge zur vorweltlichen Fauna des Steinkohlen-gebirges. Bonn 1847, p. 20.

Troschel in 1857,* the latter author using this character to separate the striated-scaled "*Amblypteri*" of Saarbrücken and Lebach, under the name of *Rhabdolepis* (*R. macropterus*, *R. eupterygius*) from their smooth-scaled associates with minute slender teeth, for which the term *Amblypterus* was retained (*A. latus*, *A. lateralis*).

An examination of the type-specimens of *Elonichthys* in the Mineralogical Museum at Halle has, however, convinced me that the genus is tenable, and that to it is referable an extensive series of Palæoniscidæ, which will include, besides Giebel's originals and several new species, various forms referred by Agassiz and other writers to *Amblypterus*, *Palæoniscus*, and *Pygopterus*. Several species also, whose position, from want of sufficient information, is somewhat doubtful, may best be placed here provisionally.

Elonichthys most closely resembles *Acrolepis* and *Rhabdolepis*, but differs from the former in the anterior covered area of the scales being reduced to a very narrow margin, and from the latter in the absence of the subopercular plate. From *Amblypterus*, as restricted by Troschel, the greater obliquity of the suspensorium, and the dentition, are obvious marks of distinction; the teeth in the true *Amblypteri* being minute, slender, without differentiated laniaries, and more meriting the title "en brosse" than those of any other genus of Palæoniscidæ. From *Palæoniscus* it is distinguished by the large size of the median fins, and by the possession of large conical teeth in the jaws, the teeth of *Palæoniscus*, as that genus must now be restricted, being small, and though, of different sizes, without conspicuous laniaries. It is also widely separated from *Pygopterus* by the structure of the pectoral fin, in which the principal rays are articulated for the greater part of their length, whereas in the last-named genus, they are unarticulated till towards their terminations; by the position of the dorsal fin, which, in relation to the anal, is further forward; and by the form of the anal, which is not produced backwards in a fringe-like manner along the lower margin of the body.

Accordingly *Amblypterus nemopterus* of Agassiz, and one of the two species included in his *A. punctatus*, must be transferred to *Elonichthys*, as well as his *Palæoniscus Robisoni*, *P. striolatus*, *P. Egertoni*, and *Pygopterus Bucklandi*. The mutual resemblances of these forms are so close that it seems almost unaccountable that they should have been placed by Agassiz in different genera; his having done so seems, indeed, to indicate the arbitrary nature of the distinction which he drew between *Amblypterus* and *Palæoniscus*, as well as his not having fully realised the nature of the essentially distinctive characters of the true *Pygopteri*.

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* "Verh. des naturhist. ver. d. preuss. Rheinlande," xiv. (1857) p. 12.

Amblypterus Portlockii of Sir Philip Grey-Egerton[*] seems to belong to this genus, as probably also does *Palæoniscus Brownii* of Jackson, judging from the figure given.[†] The same must also be said of *P. peltigerus* of Newberry,[‡] which was indeed first described by that author as an *Elonichthys*.[§]

As above defined, *Elonichthys* is pre-eminently a carboniferous genus, and well represented in strata of that age in Great Britain and other countries. The necessary restriction of the limits of the genera *Palæcniscus*, *Amblypterus*, and *Pygopterus*, and the transference of the carboniferous species formerly referred to them to *Elonichthys* and other genera (*Nematoptychius*, *Rhadinichthys*) leave the three first named without representatives in the carboniferous formation, so far as our present knowledge goes,—a conclusion which may appear at first somewhat startling to geologists, who have been so long accustomed to look upon them as characteristic of that era, as well as of the succeeding Permian.[‖]

Elonichthys nemopterus, Agass. sp.

Amblypterus nemopterus, Agassiz, Poissons Foss. vol. ii. p. 1 p. 107;
Atlas, vol. ii. pl. 46, figs. 1 and 2.

To Sir Walter C. Trevelyan, Bart., of Wallington, Northumberland, I am indebted for the loan of the original example of this species figured by Agassiz; the only other specimen, which I can with certainty identify with it, is contained in my n collection. Its principal specific characters may be summed up as follows:—

Length from 4 to 5 inches; scales rather small, their posterior margins finely denticulated, ornamented with delicate ridges, which are mostly parallel with the anterior and lower margins, some above being parallel with the upper one, and which in the hinder part of the fish, tend, on the posterior part of the scale, to become replaced by punctures; dorsal and anal fins very high in front sharply acuminated; the rays of all the fins delicate, with rather distant articulations (in the principal rays at least twice as long as broad).

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[*] Quar. Jour. Geol. Soc. vi. 1850, p. 2.

[†] Report on the Albert Coal Mine, New Brunswick, p. 52, pl. i. fig. 2.

[‡] Geol. Survey of Ohio, Palæontology, vol. i. p. 345, pl. xxxviii. fig. 1.

[§] Proc. Acad. Nat. Sc. Philad., 1856; pp. 96–100.

[‖] The strata at Saarbrücken and Lebach, containing the typical *Amblypteri*, as well as the fish-bearing beds of Münster-Appel, Kreuznach, Goldlauter, &c., in Germany, and of Autun in France, long believed to belong to the carboniferous formation, are now referred by continental geologists to the Lower Permian ("unteres Rothliegendes").

By Agassiz the rays of the dorsal fin are described as being "tous bifurqués a plusieures reprises jusque vers leur milieu." It seems to me, however, that the dichotomisation of the longer rays, as is usual in this genus, does not commence till towards their terminations. He also described the surface of the scales as being "orneé de petites rides saillantes, disposées à-peu-près comme les lignes d'accroissement en losanges concentriques plus on moins régulières et un peu obliques, de telle sorte que leurs angles aigus sont tournés vers les angles supérieur-antérieur et inférieur-postérieur de chaque écaille,"—a description which is hardly correct, as there are no striæ parallel with the posterior margin of the scale. The scales of the type-specimen are, however, in very bad preservation, the exterior ganoine layer having everywhere disappeared, and their markings can only be made out from the impressions on the counterpart. Agassiz seems also to have been much struck by its resemblance to the *Amblypterus* (*Rhabdolepis*) *macropterus* of Saarbrücken; to me, however, although the bones of the head are very badly preserved, its affinities to the following species seem so plain and obvious, that I have no hesitation in referring it to the same genus.

Geological Position and Locality. In ironstone nodules from the Lower Carboniferous shales of Wardie, near Edinburgh.

Elonichthys intermedius, sp. nov. Traquair.

Amblypterus punctatus (pars) Agassiz "Poissons Fossiles," vol. ii. pt. 2, p. 109. Atlas vol. ii. tab. 4c. figs. 3, 5, 6, 7, 8, but *not* fig. 4.

Eliminating from the three figured specimens of "*Amblypterus punctatus*" Ag., the one showing a head, and which in my last communication to the Society I have made the type of a new genus, *Gonatodus*, retaining the original specific name *punctatus*, the two remaining headless specimens seem undoubtedly to agree with each other, and also with a series of more or less perfect fishes in my own collection, which belong to a species perfectly distinct, even generically, from the first. The presence in the jaws of this species of sharp conical teeth of different sizes, as well as its entire aspect, shows that it is referable to the same genus as *Elonichthys striolatus*, (*Palæoniscus striolatus*, Agassiz) with which it is indeed closely allied.

Description.—The largest specimen in my collection measures 6 $\frac{4}{5}$ inches in length, but, as it is a little contorted laterally, it must have been originally somewhat longer; the more ordinary length is from 5½ to 6½ inches. The form of the body is fusiform, but as hardly any specimen is absolutely free from some amount of distortion, it is difficult to ascertain the proportional measurements with absolute accuracy; two very good examples, however, agree in this, that the length of the head is contained a little more than four times, and the greatest depth of the body (just in front of the dorsal fin) a little more than three times in the total length, up to the bifurcation of the caudal fin.

Many of the details of the structure of the head are clearly shown. The outer surfaces of the bones of the cranial shield display a fine tubercular ornamentation, the tubercles frequently passing into short rugæ; the superethmoidal forms as usual a blunt prominence projecting over the front of the mouth. The suspensorium is very oblique, and the gape consequently of great extent. The operculum is rather narrow, its posterior-superior angle is rounded off, the posterior-inferior one acutely pointed, its outer surface is marked with delicate and somewhat distant ridges, which mostly follow the direction of the lines of growth. The interoperculum is square-shaped and apparently similarly ornamented, though it is difficult to obtain a satisfactory view of its external surface. The exact number of the branchiostegal rays is hardly ascertainable; in many specimens the anterior one of each lateral series is distinctly shown to be much broader than the rest, and a median lozenge-shaped plate is also conspicuously present between these, and behind the symphysis of the mandible. The maxilla is of the usual form; its broad portion is ornamented with delicate raised striæ placed rather widely apart, and mostly parallel with the superior and posterior margins; they are closer and more pronounced along the dentary margin. The mandible is moderately stout, and closely striated externally with delicate ridges, which run in a direction slightly oblique to its upper border. The teeth are rarely seen, being usually covered up by the matrix, whose intense hardness defies all attempts at working them out; in some specimens, however, a few are exhibited both on the mandible and maxilla. They are conical, sharp, and incurved, and of different sizes, the largest measuring about $\frac{1}{30}$ inch in length, though in one specimen a broken tooth is perceptible which must have been larger, probably attaining about $\frac{1}{20}$ inch in length. The bones of the shoulder girdle, usually well seen, present nothing remarkable in their configuration; they are ornamented externally with closely set, wavy, branching, and anastomosing ridges.

The scales of the body are of medium size; as usual they diminish from before backwards, but not in so very marked a manner as in *Gonatodus punctatus*. Externally they are marked with oblique furrows passing into isolated coarse punctures in the posterior part of the scale. In the anterior part of the fish, and along the line of the back in front of the dorsal fin, these furrows are closer, producing sometimes a ridged appearance between them, and extend more over the surface than is the case further back, where they are shorter, and more limited to the anterior margin, so that the greater part of the exposed area of the scale is simply punctured; the furrows become, indeed, almost entirely lost on the scales of the tail pedicle. The posterior margins of the scales are finely denticulated.

The length of the pectoral fin is equal to about $\frac{2}{3}$ that of the head; it is sharply acuminated, and contains about 30 rays, which at the medial edge of the fin become very short and delicate. The first three rays on the lateral edge, which are short and do not reach the apex, seem entirely unarticulated; the articulations of the others are distant proximally, but becoming rapidly closer distally, so that over the greater part of the expanse of the fin the joints are square-shaped, becoming at length even shorter than broad. The ventrals are situated midway between the pectorals and the anal; they are also acuminate in form, each containing about 15 rays, articulated from their origins, their joints also diminishing in length from their proximal towards their distal extremities. The dorsal is situated nearly opposite the interval between it and the anal; both of these fins are very similar in shape, being triangular, very high in front, acutely pointed, and with the posterior border gently concave. Their rays appear coarse, compared with those of *Elonichthys nemopterus*, and especially with those of *Gonatodus punctatus*, so that had Agassiz only seen the fins of the latter he could not have confounded it with the present species. Each consists of about 30 rays, finely striated externally in the direction of their length; their transverse joints are conspicuously longer than broad in the principal rays, but become shorter in the posterior part of the fin, and also in the longer rays towards their extremities. The longer rays begin also to bifurcate towards their terminations, the process gradually creeping up on the posterior part of the fin till in the delicate hindermost rays it takes place about their middle,—a very general characteristic of the fins in the species of this genus. The caudal is very powerful; the rays of its lower lobe are divided by rather distant articulations, and dichotomise towards their extremities; those of the upper lobe divide about their middle, and are characterised by the extreme closeness of their articulations, the joints being rather shorter than the rays are broad. The fulcra of all the fins are minute and closely set.

Remarks.—The close resemblance which this species bears to *E. striolatus* is quite apparent; their specific distinction is also sufficiently pronounced. In *E. striolatus* the articulations of the principal rays of the dorsal and anal fins are much closer; the dorsal and anal fins do not seem to be quite so sharply acuminate in form; the laniary teeth are larger; the scale ornament, though belonging to the same general type, is more delicate in character. To *E. nemopterus* its resemblances are also very strong; it differs, however, from it in the greater coarseness of the fin rays, with closer articulations, this being especially marked in the case of the pectoral, as well as in the nature of the scale ornament. The aspect of the fin rays is somewhat intermediate between their condition in the two other species named, hence the specific term which I have adopted for the present fish.

Geological Position and Locality.—In ironstone nodules from the Lower Carboniferous shales of Wardie, near Edinburgh. In the collection of James Linn, Esq., there is also a specimen from Dechmont in Linlithgowshire, in very indifferent preservation, but which I am inclined to refer to the same species.

3. Note on Clapeyron's Formula. By Professor Dewar.

4. Note on the Specific Gravity of Ocean Water. By J. Y. Buchanan.

During the cruise of the Challenger the specific gravity of the surface water was observed daily while at sea, and that of water from the bottom and intermediate depths as often as opportunity offered. These observations were almost entirely confined to the region between the parallels of 40° N. and 40° S. latitudes, which, including as it does the regions of the equatorial calms, of the trade winds, and to a certain extent of both the northern and southern westerly winds is decidedly the most interesting field for such investigations. Many valuable observations have been made by other observers, and I have availed myself more especially of the series of observations made by Lenz during Kotzebue's voyage to the Pacific and back. His observations were almost entirely confined to the surface water.

The density of sea water depends on the amount of salt which it contains, on its temperature, and on the pressure to which it is subject. Away from the disturbing influences of the shore, the water of the ocean is subject to comparatively slight variations in saltnesss, so slight indeed that different waters may without sensible error be assumed to be equally affected by changes of temperature and pressure. As the remarks which are offered in the present paper have reference to the salinity or concentration of the water, the specific gravities have all been reduced to their value at 60° F., the unit being that of distilled water at its temperature of maximum density. The pressure is that of one atmosphere.

There are two meteorological causes which affect the saltness of the sea; the one is the formation of vapour and the precipitation of it as rain, and the other is the freezing of the water and melting of the ice thus formed. Vast quantities of solid matter, chiefly carbonate of lime and silica, are removed from the water by living organisms, and these substances form the chief components of the solid matter brought down in solution by rivers. Although this is a very important cause of the transmigration of rock, it does not sensibly affect the specific gravity of the sea-water, because only a very limited amount either of silica or of carbonate of lime can be in solution at any one time. The general distribution of saltness, as indicated by the specific gravity of the water at a common temperature of 60° F. at the surface, is shown on a chart on which localities of equal saltness are connected by lines, and the spaces between successive lines coloured distinctively. The vertical distribution is shown in diagrams in sections along certain lines of route, the depths at which the same saltness is observed being also connected by lines. The variations of saltness at the surface on the lines of section are further shown by curves having for ordinates the reduced specific gravity, and for abscissæ degrees of latitude or longitude according to circumstances.

These diagrams show at once how great are the differences in the saltness of the water in different depths and at different localities on the surface, and it must be remembered that we are here considering *bona fide ocean* water, litoral waters so much influenced by the drainage of the land being reserved for separate treatment. Looking at the chart we see that specific gravities above 1·0270 occur in three localities only—namely, in the trade-wind regions of the North and South Atlantic, and of the central South Pacific. In the North Pacific the specific gravity was nowhere as high as 1·0265, whereas large tracts both of the North and South Atlantic reached 1·0275. The greater part of the year 1873 was passed in the North Atlantic, and the central part was crossed on the homeward voyage. On the eastern side and in the track of outward bound ships, the maximum of 1·02763 was reached in latitude 24°. In the central part the maximum was 1·02768 in latitude 25°, and in the western part it was 1·02743 in 27°30′; so that between the meridians of 20° W. and 70° W. the line of maximum saltness would appear to run nearly due east and west, with a slight northerly inclination towards the west. In the central South Atlantic the maximum of 1·02768 was observed in latitude 17°26′, and in the western part 1·02783, in latitude 15°15′, so that here also the line of maximum saltness would appear to have a slight northerly trend. Between these two lines the specific gravity falls off until a minimum of 1·0260 is reached at about 3° N. latitude, and following the equator from east to west the specific gravity appears to increase pretty steadily. In the Central Pacific the maximum saltness is observed in latitude 18° S., where the specific gravity is 1·02725; in the north the maximum is 1·0263, in latitude 22° N. The minimum of 1·02488 was found in latitude 7°26′ N. In the Western Pacific the series of observations was not so continuous, there being a considerable gap between latitude 15° S. and 2 S.; the northern portion, however, is complete. Here, again, the absolute maximum is in the south, reaching 1·0268, in 22°30° S.; in the north the maximum is 1·0263 in 22° N., but the dense water of the Southern Pacific penetrates into the northern part of the ocean, giving another and higher maximum of 1·0266 in lat. 2′ N. The actual maximum observed was 1·0242 close to Humboldt Bay, in New Guinea. Other very low specific gravities were observed occasionally even at considerable distances from the coast; but the fact that there was frequently drift-wood floating about, and that the light water was confined to a stratum of 10 to 20 fathoms thickness, showed that we had here to do with shore water. If the general run of the curve be followed, the oceanic minimum will be found to occur about 10° N., marking 1·0258.

The saltness, which increases on both sides of the equator until a maximum is reached in the trade wind districts, decreases again towards the poles and the whole of the ocean to the southward of latitude 40° S., has a specific gravity below 10260, and for the greater part below 1·0255. In the neighbourhood of ice in summer great and sudden variations are frequently observed, the specific gravity amongst loose park ice frequently falling as low as 1·0240.

Confining our consideration, then, in the meantime, to the surface, we find that the maxima in the Southern Pacific and Atlantic are situated nearer the equator than those in the northern parts of these oceans; that the maxima in the Atlantic are higher than the corresponding ones in the Pacific; and that the *maximum maximorum* occurs in the South Atlantic about latitude 15° S. The equatorial minima in both oceans are to the northward of the equator; and from all these facts we see how intimately the concentration of the sea-water is connected with the distribution of the trade winds.

If we turn now to the consideration of the vertical distribution of density, we see that the influence of atmospheric conditions does not cease when we leave the surface. In the trade-wind districts we find the excessive concentration extending to greater or less depths, according to the ocean which we are considering, and, as might have been expected, this effect is much more sensible in the Atlantic than in the Pacific. In the Atlantic, again, the amount of concentration is much greater in the northern than in the southern portion, while in the Pacific the reverse is the case. The great amount of concentration in the North Atlantic is doubtless due chiefly to the form of its basin. The south-east trade which blows with a preponderating force drives constantly fresh supplies of water in to the northern basin, where a large portion of it is kept constantly circulating by the prevailing winds in a region where the atmosphere is ever greedy of moisture. This circulation, assisted by the yearly oscillations of temperature, bring about a constant concentration of the deep waters, and at the same time, as I have pointed out, a heating of the deeper waters by convection. The effect of this concentration of the water is to render it heavier than an equal bulk of water further to the south, and consequently it forces its way southwards along the bottom. This under outflow from the North Atlantic was conjectured by Captain Tizard from a consideration of the temperature of the deep water of the western basin of the South Atlantic, and a glance at the vertical section of the Atlantic is sufficient to show the great likelihood of its existence. From a consideration of this section it would appear probable that at some position about 40° north latitude a tolerably uniform density of from 1·0265 to 1·0270 will be found, and, indeed, in the neighbourhood of the Azores the following bottom specific gravities were observed:—

Depth in fathoms, . .	1675	1000	900	750
Specific gravity at 15·56, .	1·02670	1·02693	1·02691	1·02679

Although we find that in all tropical and temperate regions the temperature of the water is highest at the surface, and decreases as the depth increases, we do not by any means invariably find the highest specific gravity at the surface, nor does it decrease regularly with increasing depth. The water is usually least salt at between 500 and 1000 fathoms from the surface. The maximum specific gravity at any locality is frequently observed at depths of from 25 to 150 fathoms from the surface, and it is not unusual to find besides the normal minimum between 500 and 1000 fathoms, a second minimum at 25 or 50 fathoms from the surface. These maxima and minima are caused in some cases by currents, and in others are the evidence of past wet or dry seasons. In the Southern Ocean the surface specific gravity varies between 1·0240 and 1·0250, being nearly constant at 1·0250 in localities free from ice; it increases, however, with the depth, the bottom specific gravity being usually 1·0255 or 1·0256.

The following Gentlemen were elected Foreign Honorary Fellows, to fill up the vacancies caused by the death of Adolphe Theodore Brongniart and of Christian Gottfried Ehrenberg:—

ALPHONSO DE CANDOLLE, Geneva.
Professor CARL GEGENBAUR, Heidelberg.

The following Gentlemen were elected Ordinary Fellows of the Society:—

JOHN MURRAY.
WALTER NOEL HARTLEY, Chemical Demonstrator, King's College, London.
WALTER WELDON, F.C.S., London.

Monday, 29th January 1877.

Professor KELLAND, Vice-President, in the Chair.

The following Communications were read:—

1. Note on the Manganese Nodules found on the Bed of the Ocean. By J. Y. Buchanan.

The manganese nodules occur in greater or less quantity all over the ocean-bed, and most abundantly in the Pacific. They occur of all sizes, from minute grains to masses of a pound weight, and even greater, and form nodular concretions of concentric shells, round a nucleus, which is very frequently a piece of pumice or a shark's tooth. Their outside has a peculiar and very characteristic mammillated surface, which enables them to be identified at a glance. When freshly brought up they are very soft, being easily scraped to powder with a knife. They gradually get harder on exposure to the air.

The powder, heated in a closed tube, gives out water which reacts alkaline, and has an empyreumatic odour. Heated with strong hydrochloric acid, it liberates abundance of chlorine, and the residue which remains is white, consisting of silica, clay, and sand, the sand being the same as is found in the bottom mud from the same locality. Their composition varies greatly, different nodules containing different quantities of mechanically admixed mud, and the number of different elements found in them is very large. Copper, iron, cobalt, nickel, manganese, alumina, lime, magnesia, silica, and phosphoric acid have been detected in a large number; but I have not as yet been able to make a completed analysis of any of them. I have, however, made a few determinations of the most important component substances. For this purpose the outside and densest layers of the nodules were selected, and portions of them pulverised and dried for ten or twelve hours at 140° C. The amount of chlorine liberated on treatment with hydrochloric acid was determined by Bunsen's method, and the iron was determined by titration with stannous chloride. The samples analysed were from four different localities.

Nos. 2, 4, and 5 were from the same place, No. 2 being the matter collected round a shark's tooth as nucleus; Nos. 4 and 5 being the outside rinds of ordinary nodules.

The results are given in the following table, the numbers being in many cases the means of several observations :—

Locality.		No.	A	B	C	D	E	F	G	
Lat.	Long.		Insoluble Residue.	O	MnO_2	MnO	Fe_2O_3	Al_2O_3	H_2O	Na_2O
13° 52′ S.	149° 17′ W.	2	17·55	6·13	33·30	27·18	...	...	...	...
...	...	4	15·30	5·92	32·23	...	23·86	...	...	...
...	...	5	15·30	6·49	35·28	...	24 85	...	10·2	...
37° 52′ N.	160° 17′ W.	6	36·24	6.49	24·41	...	20·16	3·83	7·70	5·98
42° 42′ S.	134° 10′ E.	7	17·98	7·54	41·11	33·53	18·04	2·55	7·31	...
22° 21′ S.	150° 17′ W.	8	21·74	5.19	28·20	...	24·52	7·67	8·54	8.5

A is the residue which remains undissolved after treating the mineral with strong hydrochloric acid, evaporating to dryness and redissolving. In No. 5 it contains 85·16 per cent. silica, and in No. 6, 82·27 per cent.

B is the available oxygen determined by Bunsen's method.

C is the MnO_2 equivalent to the available oxygen.

D is the MnO found by weighing as Mn_3O_4.

E is the Fe_2O_3 found by titration with $SnCl_2$.

F is the alumina found by subtracting the Fe_2O_3 found in E from the weight of the precipitate with acetate of soda.

G is the water expelled on ignition; it is obtained by deducting two-thirds of the oxygen found in B from the loss of weight by ignition.

It will be seen from the results given in the above table that the nodules from different localities vary greatly in composition, though in the same locality they have similar composition, irrespectively of the nature of the nodules. The insoluble residue contains, besides silica and clay, sand of the same mineral nature as is found in the bottom at the same locality. The manganese is present wholly as MnO_2, and the iron as Fe_2O_3. In No. 6 there is 3 per cent. of cobalt; this metal, along with copper and a little nickel, is present in all of them. Zinc was not found in any of the above specimens.

2. Note on the Measure of Beknottedness. By Prof. Tait.

In drawing the various closed curves which have a given number of double points, I found it desirable to have some simple mode of ascertaining whether a particular form was a new one, or only a deformation of one of those I had already obtained. Of course the *schemes* (as described in my former paper) contain the desired information, but it may sometimes be difficult to obtain in this way; for, when the number of intersections is large, we may have to change the crossing which is taken as the initial one several times before we hit upon the same notation for like crossings (if such exist) in the two schemes compared. And the methods of deformation already given often present their results in forms so distorted that it is not easy at once to recognise their identity with other drawings of the very same curves.

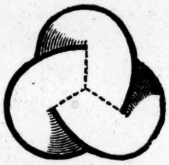
The method of treatment described in my paper, which depends upon the study of the *plait*, supplies (by the + and − signs *over* the various crossings) exactly the sort of information we require, though it may leave ambiguities. But some simple mode of applying it is requisite.

I first tried a modification of the process (formerly described) of going round the curve and pitching a coin into each field or cell as it is reached. To make the required distinction between crossing *over* and crossing *under*, we may suppose the two coins to be of different kinds,—silver and copper for instance. Let the rule be:—silver to the right when crossing over, to the left when crossing under. Then, however the path be arranged, of the four angles at each crossing, one will have no coins, the vertical or opposite corner will have *two* silver or *two* copper coins, the others *one* copper or *one* silver coin each.

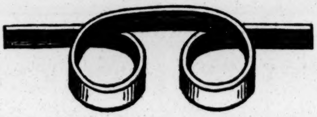
It is easily seen that a reversal of the direction of going round leaves the single coins as they were, but shifts the pair of coins into the angle formerly vacant; also that in the deformed figures the circumstances are exactly the same as in the original. Hence we may divide the crossings into silver and copper ones, according as two silver or two copper coins come together. And the excess of the silver over the copper crossings, or *vice versâ*, furnishes an exceedingly simple and readily applied test (not however, as will soon be seen, in itself absolutely conclusive of identity, though absolutely conclusive against it), which is of great value in arranging in family groups (those of each family having the same number of silver crossings), the various knots having a given number of intersections.

I soon saw that this process, so limited, was intimately connected with that required for the estimation of the work necessary to carry a magnetic pole along the curve, the curve being supposed to be traversed by an electric current, and it occurred to me that we might possibly obtain a definite measurement of beknottedness in terms of such a physical quantity: as it obviously must be always the same for the same knot, and must vanish when there is no beknottedness. The measure may be made more complete by recording the numbers of non-nugatory silver and copper crossings separately, with the number to be deducted as due merely to the *coiling* of the figure. I shall recur to this point later.

When unit current circulates in a circuit, the work required to carry unit pole once round any closed curve once linked with the circuit is $\pm 4\pi$. Instead of the current we may substitute a uniformly and normally magnetised surface bounded by the circuit. The potential energy of the pole in any position is always measured by the spherical opening subtended by the circuit; but its sign depends upon whether the north or south polar side is turned to the pole. Hence there is no potential energy when the pole is situated in the plane of the circuit but external to it, and the value is $\pm 2\pi$ when the pole just reaches the plane of the circuit internally. Gauss gave from these results the value of a remarkable double integral extending over each of any two closed curves linked together in space. Clerk-Maxwell (*Electricity*, § 422) has shown that this integral may vanish even for a complex linking of the two circuits; and a similar difficulty is met with in the single circuits with which we are now dealing, so that a special set of rules must be made for determining the beknottedness in terms of the silver and copper junctions. But the difficulty just mentioned leads, as will be seen, to a very curious result.

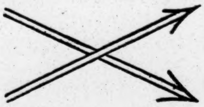


To construct the magnetised surface which shall exert the same external action on a pole as a current in any given closed circuit does, we may either suppose a surface extending to infinity in one direction (say, for definiteness, upwards from the plane of the paper), and having the circuit for its edge; or we may form, as in the figure, a finite autotomic surface of one sheet, having the circuit for its edge. The only difficulty in estimating the work lies in the definite statement of how the pole is to move along the curve itself. For, if its path screw round the curve, $\pm 4\pi$ must be added to the work for each complete turn. As an illustration, suppose we bend an india-rubber band coloured black on one side, as in the figure, so that the black is always the concave surface, we find on pulling it out straight that it has no twist. If both loops be made by *over*laying, when pulled out it becomes twisted through two whole turns. This is an instance of the kinematic principle that spiral springs act by torsion.

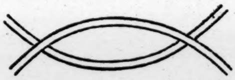


Perhaps the most simple definite condition is that which I first employed, viz., to make the pole move along the curve, keeping always in the osculating plane and on the convex side. But we have then to arrange beforehand what is to be done at a point of inflexion.

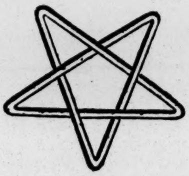
A practical rule, however, may easily be given from the consideration of the magnetised surface above mentioned. Go round the curve, marking an arrow head after each crossing to show the direction in which you passed it. Then a junction like the following gives + 2π at each time of crossing; or, if we use the infinite surface spoken of above, it gives + 4π for the upper branch, and nothing for the lower (which, on this supposition, does not pass through the magnetic sheet). Change the crossing from *over* to *under*, and these quantities change sign. The junction figured above would, in our first illustration, be a silver one. But a still simpler process is to go round putting a dot to the *right* after each crossing *over*, and *vice versâ*. Silver crossings have two dots in one angle; copper one in each of two vertical angles.



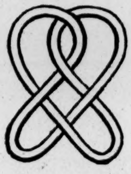
Now, in order that our rule may be such as to give no work where there is no beknottedness, we must make the required expression such as to vanish whenever all the intersections are nugatory. Those which are nugatory only in consequence of their signs are in pairs, silver and copper, and will take care of themselves, as we see by special examples like the following, in which the reversal of one of the directions simply reverses the signs. Hence the only part to correct for is that depending on the number of whole turns, and the sketch of the india-rubber band above shows that the work at the vertex of each such partial closed circuit is simply not to be counted—*i.e.*, that the ± 4π, which would be reckoned for each crossing by our rule, is to be considered as made up for by the corresponding screwing of the pole round the curve.



To illustrate the application of this process, let us consider again the two distinct forms with five non-nugatory intersections



1.



2.

(the first being a modified form of the "pentacle," the second, fig. 6

of my paper, which, for the comparison below, must be supposed to have all its signs changed) whose schemes are, respectively—

$$\begin{array}{cccccccccc|l} - & - & - & - & - & - & - & - & - & - & \\ A & D & B & E & C & A & D & B & E & C & A, \\ - & + & - & + & - & + & - & + & - & + & \end{array}$$

$$\begin{array}{cccccccccc|l} - & - & - & - & - & - & - & - & - & - & \\ A & D & B & E & C & A & D & C & E & B & A. \\ - & + & - & + & - & + & - & + & - & + & \end{array}$$

The lower signs refer to over or under, the upper to the electromagnetic work, or to the silver-copper distinction. These two instances, in which both series of signs are absolutely identical, each with each, show at once that we cannot take the two sets of signs alone as fully descriptive of the knot.

To determine the electromagnetic work for any knot, we must divide the scheme into independent circuits, no one of which includes a less extensive one; and omit from the reckoning the work for the terminal of each such circuit, and for each of the intersections which is not included in any one of the separate circuits. The particular closed circuits chosen do not affect the final result, as is easily seen by thinking of the various deformations of each figure.

In the first of the two schemes above there is but one independent non-autotomic circuit, which may be taken as

A D B E C A.

In this all the intersections are included, so that the whole work is to be found by leaving out that for A only; *i.e.*, it is -16π.

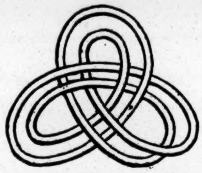
But in the second scheme we may take the two circuits

B A D B and C A D C,

and E is not included in either. Hence we must leave out of count the work for B, C, and E; and thus the whole work is only -8π.

In fact, the figures show that to untie the first knot we must not only have the signs such that we can slip off B and E, but also C and D, *i.e.*, *two* signs must be changed; while the second loses all its beknottedness if A and D could be got rid of, that is if *one* sign only be changed. This is an instance in which the estimate by the electro-magnetic process exactly agrees with the result of simpler considerations. And it is probable after all that the true measure of beknottedness is the smallest number of signs in a scheme which must be altered in order that the wire may cease to be knotted.

It will be found that the alteration of five signs is sufficient to remove the knotting from the annexed figure, and the stages of operation of the various modes of reduction show that this form can be regarded as made up of simpler knots intersecting one another on the same string. In such a case it is not easy to give a strict definition of the beknottedness in any other way than by defining it as the smallest number of changes of sign which will take off all the knotting. For the separate knots are virtually independent, and to change *all* the signs in any one of them does not in every case necessarily simplify the knot. Uncorrected the work is $-13 \times 4\pi$. Corrected it is $-10 \times 4\pi$, which agrees with the removal of the beknottedness by change of *five* signs only.



If the sign of the one unsymmetrical crossing be altered, four changes of sign will suffice; for the uncorrected work is $-11 \times 4\pi$; corrected it is $-8 \times 4\pi$, corresponding to four changes of sign.

The various modes alluded to in my paper of adjusting the (lower) signs so that there shall be no beknottedness, follow at once from these remarks. For we may make all the free letters in each circuit +, save those which we have taken, *in pairs*, in some previous circuit.

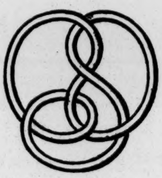
This test, though extremely useful as above explained in classifying knots with the same number of intersections, is not fully descriptive of a knot, being ambiguous whenever there is more than one class of knots with the same number of intersections and with the same excess or defect of silver as regards copper crossings. This consideration, which promises to clear up some obscure and difficult parts of the subject, has led me to some very curious results. The most important of these is when a knot, whatever be its number of intersections, has equal numbers of silver and of copper crossings, or when the uncorrected expression for the electro-magnetic work vanishes. Thinking of this in connection with the fact that a change from right-handed to left-handed in a knot simply changes silver to copper, or *vice versâ*, *i.e.*, reverses the sign of the electro-magnetic work, I was led to see that there is a class of knots which are *capable of being changed from right to left-handed, without change of form*, by the ordinary processes of deformation. Of course this implies that there is a mode of interchanging the letters, two and two, in the scheme, so that their order remains unaltered; or, what comes to the same thing, that we shall get exactly the same scheme (signs not included) by taking either of two different crossings as A, and lettering as usual from it in the same direction round the curve.

It will be readily found by trial that this can be done with the only forms which have four valid intersections—as they are figured in my former paper—if only the wire or cord be so twisted about that, while the *form* is preserved, the junctions B, D be brought into the positions relative to the figure which were formerly occupied by A, C. For the scheme

$$\begin{array}{ccccccccc} + & - & - & + & + & - & - & + & \\ A & D & B & A & C & B & D & C & \mid A \\ - & + & - & + & - & + & - & + & \end{array}$$

remains the same, so far as the letters are concerned, if we keep the same cyclical order of letters, but write A for B, B for C, &c. In estimating the electromagnetic work, by the rule above, we find we may leave out either A, C, or B, D. So that the work is $\pm 8\pi$, *i.e.*, one degree of beknottedness.

The following case, with six intersections, is very instructive. Either figure is formed from the other by throwing the lower coil over the top of the whole.



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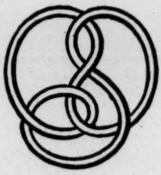

4

It will be seen that each of these forms may be regarded as a simple loop passed unsymmetrically through a simple knot of three intersections (figures 1 and 3 of my former paper), and that the knot and loop are interchangeable between two groups of three intersections. The knot is right-handed when transferred to the second group; left-handed in the first. But the figures plainly show that they may also be regarded as a right and a left-handed simple knot having a part common to each, so that neither can be pulled tight, subject to our convention that there shall be nothing higher than a double point. And *here* the peculiar difficulty associated with the amphicheiral forms comes in; for, in estimating the electromagnetic work, we find we must leave out one copper and one silver junction—the result being $+8\pi - 8\pi$. This is to be treated as $\pm 16\pi$ (or two degrees of beknottedness) because the portions with different signs belong to what are, virtually at least, two separate knots.*

The possibility of such *amphicheiral* forms is obvious from one of the first illustrations in my former paper; where we have only to suppose the irrelevant crossing removed, and one of the separate simple knots (which are both right-handed in the cut) made left-handed. But I was not at first prepared to find this property in any knot not separable into detached, self-contained portions; so that it is possible that some of my former statements may require modification.

It may be well to notice that when, in a slight variation of the arrangement just described, the loop passes symmetrically through the simple knot, we have another six-crossing form, very much resembling the last, but which is essentially not amphicheiral. It is figured below in one of its forms—the others may be got by deformation—and the schemes of the two kinds are appended for comparison.

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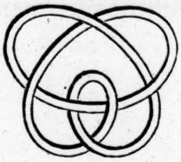
* *Feb.* 19.—This is not correct. There is but *one* degree of beknottedness, for the two knots are not "virtually separate," as they have a part in common, while one is right-handed and the other left-handed. In fact, the figures above are mere transformations of the last cut in my former paper —which is shown to be capable of being opened up by a single change of sign. This can be done in the figures above, at either end of the lower coil where it forms part of the external boundary. But if, without altering the outline of the figure, we change all the signs in either of the two component knots, so as to make them both right-handed, or both left-handed, the whole acquires the double degree of beknottedness wrongly assigned to it in the text. But it has now continuations of sign, and in virtue of these it happens to be reducible. In fact, when we make it into a clear coil after these changes of sign it becomes the pentacle (fig. 1 above), the only knot with fewer than six crossings which possesses, as we have seen, *two* degrees of beknottedness. I stated in my first paper, that when the signs in any nonnugatory arrangement are alternately + and − the cord "is obviously as completely knotted as its scheme will admit of." This completeness must



5

Figs. 3 and 4	A*D*B*E*C*A*D*F*E*C*F*B* | A.
Fig. 5	A*D*B*E*C*F*D*A*E*C*F*B* | A.

It will be seen that the sole difference between the amphicheiral knot and that last figured, lies in the inversion of the positions of A and F in their *even* places in the scheme.

It appears, then, that none of these abbreviated methods, however useful as temporary aids to classification, can take the place of the scheme in fully describing the form of a knot and in measuring the amount of beknottedness in general. Especially is the scheme required in order to calculate the beknottedness in terms of the electromagnetic work. And this conclusion might, I think, have been inferred from the prominent part which the arrangement of the letters in the even places plays in determining the form of a knot; an arrangement of which only traces are left when we substitute the sign of the work at each junction for the letter attached to it, thus losing all control of the amount to be added or subtracted on account of the mere number of coils.

[*Added Jan.* 27*th.*]—Professor Clerk-Maxwell, to whom I sent some of the above results (and to whom, as well as to Sir W.

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be understood of what may be called *Knottiness*, not of *Beknottedness*. For it has just been shown by a particular case that we can occasionally increase the degree of beknottedness, while diminishing knottiness, *i.e.*, losing crossings by so altering their signs as to make some of them nugatory. The point thus raised, *i.e.*, the distinction between Knottiness and Beknottedness, is a very troublesome and delicate one, and is obviously related to several of the difficulties pointed out in the present paper.

Thomson, I am indebted for various hints, usually in the especially valuable form of criticisms and reasons for doubt), has lately called my attention to a paper by Listing, of date 1847, part of which is devoted to the subject of knots. I have this morning obtained it from the Cambridge University Library, but havé not yet thoroughly read it. As was to be expected, I find that the author has anticipated some of the contents of my papers; and he mentions at least one very curious fact, which I had thought possible, but had not observed, though it is very directly connected with one of the results of the present note. He virtually shows, by giving a particular case, that the method of deformation which I employ does not always give all possible forms of a completely knotted wire. I believe that this depends on the fact that a part of the scheme is amphicheiral. I propose to give the Society an account of Listing's method and results on the earliest opportunity.

3. Note on the Effect of Heat on Infusible Impalpable Powders. By Professor Tait.

Several years ago Professor Dewar gave me a specimen of silica in a state of exceedingly minute division, which had been produced in Dr Playfair's laboratory in the preparation of fluosilicic acid. I noticed at the time how much its great mobility is increased by heating—so that it behaves almost like a liquid. And I fancied that I observed close to the surface a thin stratum of what might by the same analogy be called a vapour; consisting of particles thrown up and falling back again, like the little drops thrown up at the surface of soda-water. I was inclined to ascribe these phenomena to heat directly—supposing that the particles were fine enough to behave, though in a very imperfect way, as the kinetic theory assumes the particles of a gas to behave. However this may be, the extreme mobility of such powders when heated on a platinum dish; and the fact, noticed by chemists, that a bath of calcined magnesia is capable of propagating waves when heated; seem to show that valuable results might be obtained by seeking for evidence of inter-diffusion as the result of experiments made by very long-continued heating of vessels containing fine silica and magnesia originally in separate strata. I have brought this before the Society in the hope that (as it can hardly be classed as a laboratory experiment) some of the Fellows, who may have access to a suitable furnace which is in activity the greater part of the year, may be induced to give the experiment a fair trial.

4. Abstract of Additional Memoir on the Parallel Roads of Lochaber. By D. Milne Home, LL.D.

Mr Milne Home stated that he had been induced to resume his researches in the Lochaber district, in consequence of a lecture, in June last, by Dr Tyndall of London, at the Royal Institution, Albemarle Street, and subsequently printed and published. In this lecture, Dr Tyndall controverted the detrital theory of lake barriers, and advocated the glacier theory;—supporting his views by a reference to observations which Dr Tyndall said he had recently made on a special visit to the Parallel Roads.

Mr Milne Home said that on account of Dr Tyndall's reputation as a man of science, and his great knowledge of glacier action, he had thought it right to reconsider his own opinions, and go back to Lochaber with Dr Tyndall's lecture in his hand.

During this recent visit, Mr Milne Home said he had obtained farther information bearing on the question, which he hoped might be deemed sufficiently important to be communicated to the Society.

He would first mention the import of this information, and then offer remarks on Dr Tyndall's lecture.

After recapitulating the grounds on which he had in his last Memoir suggested that the barriers of the old lakes had been composed of the natural detritus of the district, he mentioned the following facts, in corroboration of these grounds—

1. He specified several additional cases of lakes, in Stratherrick (a district not far from Lochaber), now kept in by detrital blockage, and which, owing to a partial lowering of this blockage, had subsided from higher levels,—these higher levels being indicated by horizonal lines of cliff.

2. He specified the occurrence of thick beds of detritus, consisting of clay, gravel, and sand, along the hills between Stratherrick and Strathspey, at levels of from 2000 to 2500 ft. above the sea,—heights far greater than the sites of the barriers of the old Lochaber lakes.

3. He next indicated on a map the probable sites of the barriers by which the lakes of Glen Roy and Glen Spean had been respectively confined, till the barriers were broken down, and gave reasons for fixing on these sites.

4. While in his last Memoir he had attributed the breaking down of the barriers to the agency of the rivers discharging from the lakes, and also of streams flowing upon the barriers from the adjoining mountains, he now added another probable cause of injury, to the barriers at the mouths of Glen Spean and Glen Glusy in the agency of the sea, there being probable evidence that when these barriers existed, the sea was standing at a level of about 500 or 600 feet higher than at present; and that the blockages of these two glens were so near the sea, when at that height, as to form sea-cliffs.

He explained that this evidence consisted, *first*, in the occurrence of sea shells, said to have been found at two places outside the Glen Spean barrier, in beds of clay from 200 to 500 feet above the sea, by persons whose trusworthiness he had ascertained; and, *second*, in the existence of a series of terraces, in the district about Spean Bridge, from 400 to 500 feet above the sea, which he considered to be marine.

5. Mr Milne Home then adverted to the grounds on which the glacier theory rested, and stated that by another visit made last autumn to Corry N'Eoin and Glen Treig, the only two valleys, where a glacier is supposed to have existed, when Glens Spean and Roy were occupied by lakes, he had discovered that these valleys, instead of being then filled with ice, must have been occupied by water, viz., the water of the Glen Spean Lake; inasmuch as there are beach lines round the whole of L oc Treig, and also at the mouth of Corry N'Eoin.

6. Referring to what had been called the Moraines of the alleged Treig glacier—viz., at Murlaggan, and in the district between the Rough Burn and Fersit,—he said that at Murlaggan

the knolls referred to were composed entirely of beds of sand and fine gravel,—manifestly deposited by water, and not brought by a glacier.

The supposed Moraines, between Rough Burn and Fersit, consist of enormous embankments of gravel, apparently sub-marine, and formed when the sea stood some thousands of feet above its present level. The boulders strewed over this district, and heaped on these embankments, had probably been brought by floating ice, in a current from the westward, flowing up the valley of Spean towards Strathspey.

7. In regard to Dr Tyndall's lecture, Mr Milne Home pointed out, that when impugning the soundness of the detrital barrier theory, Dr Tyndall had given a representation of that theory only from Sir Thomas D. Lauder's Memoirs, written fifty-nine years ago, and not from the more recently published Memoirs, which stated facts and arguments not to be found in Sir Thomas Lauder's paper. Moreover, Dr Tyndall had unfortunately misapprehended Sir Thomas Lauder's views regarding the formation of the barriers, ascribing to him views on this subject disparaging to the detrital theory, which neither Sir Thomas nor any other author entertained.

The only specific objection to the detrital theory suggested by Dr Tyndall, was the alleged absence of any traces of barriers. Mr Milne Home stated that he thought there were traces; and that, even if there were none, the want of them would be no valid objection to the theory.

With regard to the grounds on which Dr Tyndall maintained the glacier theory, Mr Milne Home stated that Dr Tyndall ought to have shown, how there could be such a difference in the climatic condition of the two sides of Spean Valley, that while the glens on the south side were filled with ice, the glens on the north side were filled with water;—this, however, Dr Tyndall had failed to do.

Mr Milne Home farther stated that though it was undoubtedly true that some traces of glacier action were visible in various parts of Lochaber, these, as it seemed to him, belonged to a period in the world's history long antecedent to the lakes of Glen Roy and Glen Spean. When these lakes existed, the water of the Spean lake penetrated into the two glens (Corry N'Eoin and Treig), supposed to have been occupied by glaciers. It was therefore a physical impossibility, that there could have been glaciers in these two glens at the time that there were lakes in Glen Roy and Glen Spean.

Even if there had been glaciers in these glens, these glaciers, to reach the position of the Glen Roy barriers, would, after emerging from their parent glens, have had to travel seven or eight miles across uneven ground, push themselves round the corners of several hills, and rise up to a level several hundred feet above these glens. If it was possible to suppose that the tongues of these glaciers could reach the required spots, the notion of their forming solid and permanent barriers at these spots seems quite untenable.

Monday, 5th February 1877.

Sir WILLIAM THOMSON, President, in the Chair.

The following Communications were read :—

1. On some Effects of Heat on Electrostatic Attraction. By Professor Tait.

2. On the Curves produced by Reflection from a Polished Revolving Wire. By Edward Sang, Esq.

When a polished thin straight wire turns on a fixed point in space, the point at which light coming from a fixed source is reflected, moves in a curved surface. In this paper the motion of the wire was supposed to be restricted to the plane passing through the eye and the source of light. The curve was shown to be of the third order, having a straight line as a symptote both ways, and to depend for its form upon a characterising angle. The interest of the subject lay chiefly in the remarkable transformations of the curve.

3. On an Ammonia-Cupric Zinc Chloride. By E. W. Prevost, Ph.D.

The following is but a short and incomplete account of a compound formed on the carbon and binding screws of a makeshift Leclanché battery. The cells employed were ordinary Bunsen elements, of which the carbon was embedded in manganese dioxide;

the substance in question grew on the brass binding screws, and as a thin layer on the carbon. The external appearance of the mass was reniform, the colour ranging from a pale blue to a dark purple and lilac tint; the portions from the carbon were of a pale green shade, and of no special shape. The quantity being small, it was impossible to make more than one analysis of each substance and that only with very small quantities. The exterior portions which showed no crystalline structure, and were efflorescent, appeared to contain less water than the interior portions, which possessed a paler colour, the composition being otherwise the same. The interior was not homogeneous, as particles of some white substance could be distinguished scattered through the mass. The dark purple and green portions, when heated to 100° C, give off ammonia and water in varying quantities (NH_3 2·02–0·87 per cent.), and left a bluish green powder, which was only slightly soluble in water or in dilute sulphuric acid, but easily dissolved with effervescence by nitric or hydrochloric acid. The original substance treated with water formed a blue alkaline solution, resembling that produced by the solution of cupric hydrate in excess of ammonia, a green residue being left undissolved, which, with potassium hydrate, yielded ammonia.

The analyses point to a formula,

$$Cu_2Zn_4Cl_6(NH_3)_4CO_3 .$$

If the carbonic anhydride be present as zinc carbonate, and in all probability the above-mentioned white particles are ($ZnCO_3$), we have present the substances,

$$(2NH_3Cu)Cl_2 + (2NH_3Zn)Cl_2 + ZnCO_3 + ZnCl_2 .$$

Also was found $(NH_4)_2(NH_3)_3Cl_5Cu_2Zn_3CO_3$.

And in another analysis, no carbon dioxide being present,

$$Cu_2Zn_2(NH_3)_3Cl_4 .$$

In contact with a portion of the substances analysed was metallic copper; this may probably account for the varying quantities of combined copper in its immediate neighbourhood, some of the results obtained indicating the presence of an ammonio-cuprous, as well as of an ammonio-cupric compound. No trace of metallic zinc could be found.

Analyses.

The following are analyses of the substances, but much reliance cannot be placed on them, as they are each from a single analysis of a very small quantity of substance.

I. Purple and lilac portion from interior, changing on exposure, giving off ammonia, insoluble in water, and depositing brown powder.

II. Pale blue, interspersed with white particles.

	I.	II.
Cu	21·67	26·77
Zn.	32·62	33·76
Cl	27·08	26·34
CO_3 (from CO_2) . . .	10·10	6·16
NH_4	6·39	6·84
NH_3	2·12	...
	99·98	99·87

4. On a Peculiarity of the Diurnal Hygrometric Curve at Geneva. By Alexander Buchan, Esq.

In a work recently published by Professor Plantamour on the climate of Geneva, the hourly means of the aqueous vapour of the atmosphere for each month is given, deduced from observations made during the twenty-seven years ending with 1875. The curves drawn from these figures are undoubtedly the most remarkable of the meteorological specialties of the climate of Geneva.

Professor Plantamour endeavours to explain the facts of the diurnal hygrometric curves at Geneva by the ascending and descending currents of air consequent on the diurnal march of the temperature. These systems of air currents serve to explain a part of the phenomena in question,—but only a part, and we think a very small part. A reference to the curves at other places is sufficient to show that this explanation is insufficient. There is another cause to which the characteristic feature of the curves is due, and that cause is the diurnal changes of the wind, which occur at Geneva, arising from its position with reference to Lake Geneva, and from the qualities of these winds as determined by the relative size of the lake.

The hourly variations in the direction of the wind show that the land and lake breezes are strongly marked—the breeze from the lake prevailing during those hours of the day when the temperature of the land is in excess of that of the lake, the land breeze during the rest of the day. In December, when the land is at no hour of the day warmer than the lake, no breeze from the lake sets in over Geneva, and also in January the breeze from the lake is but slight and of short continuance. These breezes leave their impress in a most distinct manner on the curves of the hourly variation of the vapour tension. During the winter months, when no breeze from the lake, or but a feeble one, sets in, the vapour curves show only one daily minimum, occurring about sunrise, and one maximum, about 2 P.M., whereas during the other months, from March to October, the vapour curves exhibit two daily minima, one about or shortly before sunrise, and the other from 2 to 4 P.M., and two maxima, one from 8 to 11 A.M., and the other from 6 to 10 P.M., according to season. Equally marked are the curves of the hourly variations of cloud, the maximum during the winter months occurring about sunrise, and the minimum about sunset. On the other hand, during the warm months there are two maxima and minima —the first maximum occurring about or shortly after sunrise, and the second, but by far the larger maximum, about 6 P.M., and the two minima shortly after midnight, and from 9 to 11 A.M.

The explanation of these instructive phenomena is doubtless to be found in the size of Lake Geneva, which is large enough to occasion a strong breeze during the day from the lake all round its shores. On the setting in of the breeze, the air conveyed by it, having been resting sometime previously on the surface of the lake, is necessarily moist, and while this state of matters continues the first daily maximum of vapour tension is reached. As the breeze continues, the current is necessarily drawn from higher strata of the atmosphere, and being thus but a brief interval in contact with the lake the air becomes constantly drier till the second daily minimum occurs, from 2 to 4 P.M. The breeze then gradually diminishes in force, and the air consequently becomes moister, till the maximum vapour tension of the day occurs about the time when the breeze from the lake falls to a calm, and before the land breeze springs up.

A cursory examination of the curves suffices to show that there is a close connection between their critical phases and the corresponding phases of atmospheric pressure and temperature; and the idea is suggested that in this great contribution of Plantamour's to the meteorology of Geneva, we are put in possession of data of the utmost importance as regards the relations of the vapour of the atmosphere and its movements to changes of atmospheric pressure in a way such as could be done by the observations of few observatories.

5. On Knots. By Professor Tait.

(*Abstract.*)

At the last meeting of the Society I stated that I had just procured a remarkable essay by Listing, part of which bears on the subject of knots, and that I had found in it an example of a change of form not producible by the modes of deformation I had employed.

It had for some time struck me as very singular that, though I could easily prove that (when nugatory intersections are removed) a knot in which the crossings are alternately over and under is not farther reducible, I could not prove all its possible deformations to be producible by inversions or projections of the kinds specified in my paper; but, as soon as I recognised the existence of amphicheiral forms, I saw that it was probable that they would furnish a key to my difficulty. I immediately set to work to classify the simpler of such forms; and while I was thus engaged I got the *Göttinger Studien* for 1847, in which is Listing's paper, with the title *Vorstudien zur Topologie.*

By this title Listing means *qualitative* as distinguished from *quantitative* space-relations. He commences with a study of inversion (*Umkehrung*) and perversion (*Verkehrung*),—the first being the effect of a rotation through two right angles about any axis, the second the result of reflexion in a plane mirror.

He next treats of screwing of all kinds, including twisting and plaiting.

He then applies the notion of lines winding or screwing round one another to the projection of a knot on a surface, and shows that we can thus obtain a knowledge of the relative situation of the various coils. At each crossing portions of the two branches may be regarded as small parts of lines twisted round one another. Of the four angles so formed, two vertical or opposite ones are bounded towards the *right*, the other two towards the *left*, by that line which passes *over* the other. We thus distinguish these pairs of vertical angles into right- and left-handed. [Listing uses right-handed for what we should call left-handed in screwing, but the difference is of no importance, so far as his results are concerned.]

Next he shows that perversion, but not inversion, changes right-handed into left-handed angles.

He then gives the complete knot with three intersections, and shows that when it is in a *reduced* (as distinguished by him from a *reducible*) form, all the angles in each separate mesh have a common character; but that, when it is reducible, some of the meshes have angles of both kinds. He distinguishes between the right- and left-handed forms of the reduced knot, and shows that they are not convertible into one another; also that (including external space, or the *Amplexum*) there are three meshes with two corners each, and two with three corners; one class being right-handed, the other left. And he states that the Amplex may be made to change its character from right to left by being changed from a three-cornered to a two-cornered mesh, or *vice versa.*

He points out that a loop (*i.e.*, a mesh with only one corner) does not appear in the reduced form, and then writes, as the type-symbol of the reduced right-handed knot of three intersections, the expression

$$\left.\begin{matrix} 3r^2 \\ 2l^3 \end{matrix}\right\},$$

denoting three right-handed meshes with two corners each (*Oesen*), and two left-handed meshes with three corners each (*Maschen*). [The perverted, or left-handed, form is of course represented by the same symbols, with the interchange of r and l.]

The sum of the numerical coefficients in the symbol is the number of meshes (the Amplex included), and is greater by two than the number of crossings. The sum of the products of the coefficients into the corresponding exponents gives, in each of the two parts of the symbol, double the number of crossings. These symbols contain the topologic character of any particular knotting.

Listing next points out that the reduced three-crossing knot may be obtained by three half-turns about one another of two originally parallel cords whose ends are afterwards joined into one ring, and that the character depends upon the direction of the torsion.

He proceeds to give a symmetrical knot, with seven crossings, in two different reducible, and one reduced, forms. The reduction of the first gives the three crossing knot, that of the second the four forms of the essentially non-clear coil of five intersections. (Figs. 6-9 of my first paper.)

Their common symbol he writes as

$$\left.\begin{matrix} 2r^4 + r^2 \\ 2l^3 + 2l^2 \end{matrix}\right\} ,$$

and he points out that, in this case, the Amplex belongs to each in succession of these four kinds of meshes.

He then states that the symbol

$$\left.\begin{matrix} 2r^4 + 3r^2 \\ 2l^5 + 2l^2 \end{matrix}\right\}$$

gives five different reduced forms, each with seven crossings; while the symbol

$$\left.\begin{matrix} r^5 + 3r^3 \\ l^4 + 2l^3 + 2l^2 \end{matrix}\right\}$$

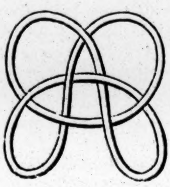
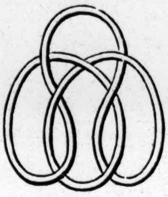
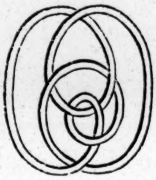
has six distinct forms.

But he adds the following extremely important remark:—"In certain cases one symbol is equivalent to another, so that the reduced forms of the one can be transformed into those of the other." He states that this is the case with the last written symbol and the following:—

$$\left.\begin{matrix} 2r^4 + 2r^3 \\ l^4 + 2l^3 + 2l^2 \end{matrix}\right\}$$

which has five reduced forms.

Thus there are, in all, *eleven* reduced forms of these kinds, all equivalent to one another, and all having seven crossings. The following are his figures: the first and second belonging to the former of the two equivalent symbols, the third to the latter—

He concludes this part of his Essay by saying that "these examples (confined as they are to single, closed lines), and the remarks made upon them, serve to show that the fundamental conception of twisting of lines is capable of being applied to the most complex space relations."

It may be added that these very elegant and important results are given as statements merely, without any hint of how they were arrived at, or how they may be extended. In fact brevity has been sedulously studied, for all that is given about knots forms a comparatively small part of the whole of Listing's extremely valuable, but too brief, Essay.

The rest treats, rather more fully, the whole subjects of inversion and perversion, screws of various kinds, plaiting and twisting, (with their applications to vegetable spirals, &c.), the numbers of lines joining given sets of points, the extensions of the meaning of the word *Area*, &c., &c.

The above abstract, which contains almost all of Listing's remarks on knots, shows that he has long ago anticipated a very great deal of what I have lately sent to the Society. For myself, however, I may say that I have had to learn only two things (about *knots*) from Listing, viz. (1), a special case (which will be examined immediately), in which two forms are equivalent, though not transformable into one another by the methods given in my first paper; and (2), to value more highly than I had hitherto done the method of classifying forms by the numbers of each kind of mesh, and the right-handed or left-handed character of each.

My first paper, as sent to the Society, was essentially confined (as indeed its title indicates) to the results deducible from a special elementary theorem,—one of two which occurred to me long ago when designing Vortex-Atoms of various forms, and which I gave to Section A at the late meeting of the British Association. The second of these theorems (as will be seen by reference to my British Association paper, *Messenger of Mathematics*, Jan. 1877), was virtually the same as Listing's division of the meshes of a reduced knot into right- and left-handed,—only I called them black and white, but as I did not see how to connect this theorem directly with the measure of beknottedness, I did not formally introduce it into my papers read to the Society. It is, of course, virtually included in the statements regarding coins thrown into the corners of cells,—for, taking the case of the silver and copper coins, the pair of left-handed vertical angles are those in which, or in one of them, there is silver—the right-handed, copper.

Nothing can be clearer than Listing's statements on several parts of the subject: it is greatly to be desired that he had made many more. Still, with a cordial recognition of the great value of all that is to be found in Listing's paper, I adhere to what I said in my last communication, to the effect that the full character of a knot cannot be learned except from its "scheme," or something equivalent to it. That the type-symbol (when such a representation is possible) is ultimately equivalent to the scheme may possibly be true,* especially when we consider that it virtually contains *two independent* descriptions of a knot (*i.e.* in terms of its right-handed and its left-handed meshes separately); that for purposes of classification it is superior is, I think, obvious, but I think it equally obvious that for the purpose of drawing the knot it is inferior. And the scheme for a reducible knot is quite as simple as that for a reduced one, while it is not easy to see what would be called the symbol of a reducible knot. Nor can I represent by

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* (*Added Feb.* 7.)—I have just found symbols for which this is not the case. The following single instance is sufficient, for the present, to show that the type-symbol is not always equivalent to the scheme. The symbol

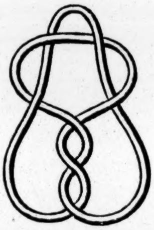
$$\left.\begin{matrix} r^4+2r^3+2r^2 \\ l^5+l^4+l^3+l^2 \end{matrix}\right\}$$

may represent either a continuous curve with 7 intersections, or a complex system consisting of a circle intersected at six points by a skewed figure of 8. I shall discuss the subject fully in a paper "On Links," which I have in preparation.

Listing's notation the double trefoil knot which has appeared in each of my papers; for, although irreducible (at least so far as I am aware), it contains several meshes which have angles of essentially different characters. Listing's avowed object was to simplify notation as far as possible. My impression is that, in one respect at least, he has carried simplification a little too far; for it cost me some little time and trouble to draw, from his type-symbol, the one knot which he speaks of, but leaves undrawn, viz., as above,—

$$\left.\begin{array}{l}2r^4+3r^2\\2l^5+2l^2\end{array}\right\}.$$

Here is one of its forms: transformation will give the four others.



In fact the type-symbol, even in this specially simple and symmetrical case, where it is much condensed, contains just as many separate typographical characters as the scheme; and I think there can be no doubt whatever that it is almost incomparably more easy to draw the figure from the scheme than from the symbol. Given the scheme, the symbol can be formed from it in a moment; while the finding of the scheme from the symbol is very troublesome. But in such a matter experience is the only guide, and I have had almost no practice in trying to draw from the symbol. Listing's type-symbol leads directly, however, to an inquiry not even suggested by the scheme; (for the latter, as I have given it, is essentially confined to a single closed curve),—viz., the forms of more than one closed curve, intersecting one another or not, which jointly divide an unlimited plane into given numbers of meshes with given numbers of sides.

The first idea of this was suggested to me when I endeavoured to draw the curve with six intersections, whose type symbol is

$$\left.\begin{matrix} 3r^4 \\ 2l^3+3l^2 \end{matrix}\right\}.$$

This symbol obviously satisfies the three numerical conditions; but, on trying to draw the corresponding figure, I found that it always came out as a species of endless chain of three separate links. One of its forms, from which the others can be found by transformation, is three circles, every two of which intersect.

Two figures of 8, linked together at each end, give the symbol

$$\left.\begin{matrix} 4r^3 \\ 2l^4+2l^2 \end{matrix}\right\}.$$

And by shifting the twist from one to the other, as explained in the latter part of this paper, the symbol may be changed to

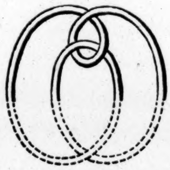
$$\left.\begin{matrix} r^4+2r^3+r^2 \\ 2l^4+2l^2 \end{matrix}\right\}.$$

I have not as yet studied the theory of type-symbols, as it differs so much from my own method; but it is obviously desirable to find the criterion by which to distinguish from one another the type-symbols necessarily denoting one closed curve, and those necessarily denoting two or more intersecting curves. It is probable that there are symbols which may represent either kind of figure. The inquiry will no doubt be found very simple, if only approached from the proper side.

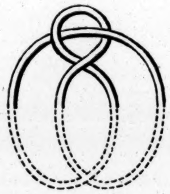
I now pass to the sole point of Listing's paper which (so far as knots are concerned) was thoroughly new to me, though not unexpected; and I shall lead up gradually to the special case which he gives, using for the purpose the properties of the amphicheiral knots mentioned in my last paper.

To apply the amphicheiral property to the production of new forms, we may begin by studying under what conditions the internal arrangements of a knot can be altered while *four* points of its contour, and *two* of the parts of the cord or wire joining pairs of these, are fixed. [The reason for the number *four* is, that when two only are employed to mark off separate parts of a knot, it is either (*a*) virtually unconstrained, or (*b*) it is divisible (and is actually divided) into two separate knots.] If, under these circumstances, changes can be made on one side of the fixtures, they will be practicable also *in whatever way these points be connected by the rest of the string*, provided always that it be not led through the amphicheiral part. Hence, if there be an amphicheiral part of any knot, it may often be transformed *in situ;* the rest of the knot being unaltered, but the amphicheiral part being made to present, as it were, another hand to the rest.

To begin with a very simple case, let us take the simplest amphicheiral form—the complete knot with four crossings—as below.



1.



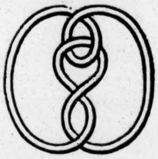
2.

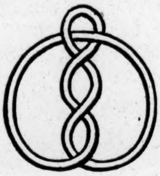
When the first of these is inverted, O being within either of the *interior* three-sided meshes, the second is produced: if O be in one of the *boundary* three-side meshes the perversion of the second is produced. But we may easily convert 1 into 2 by fixing the lower crossing (*i.e.*, by fixing a point near it in each of the four lines diverging from it, so that the dotted part remains fixed), and making the upper loop rotate so as to banish the upper crossing. Thus the upper parts of these two figures are equivalent.

And we can now suppose the (dotted) portions between the fixed points to be cut open and reknotted in any way we choose,—subject, of course, always to the rule of alternate over and under, else the knot would in general be reducible. Of course, unless we wish to study *linking*, the ends must be joined so as to preserve continuity throughout the string.

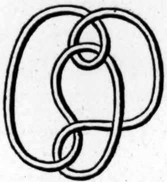
The simplest modes of joining (without additional intersections) give us at once two different aspects of the same "trefoil" knot:—with one crossing additional we have the original figures

(1 and 2) above: with two additional we have the following,— figured in my first paper.

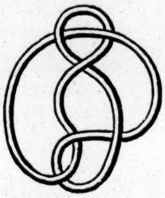




With three we have two apparently different forms,

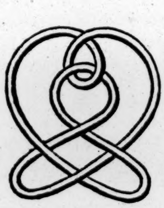


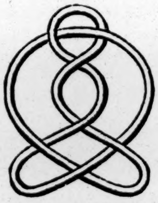
and



These are, however, only transformations of the six-crossing amphicheiral form (figs. 3 and 4 of my last paper), and are directly transformable into one another. They are, of course, unchanged if the lower part be reversed, for the upper parts are symmetrical.

But when we add four new crossings, as below, instead of the single crossing removed, we get the two equivalent figures





which are not transformable into one another by the processes of my first paper. In fact the *schemes* will be found to be incompatible, begin each where we choose. In Listing's notation their type-symbols would stand respectively, thus—

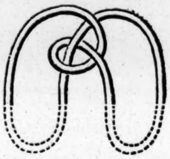
$$\left.\begin{matrix} 2r^4+2r^3 \\ 2l^4+3l^2 \end{matrix}\right\} \quad \text{and} \quad \left\{\begin{matrix} r^5+r^4+r^3+r^2 \\ 2l^4+3l^2 \end{matrix}\right.$$

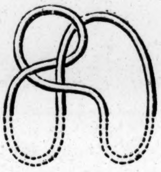
The schemes are

A*E*B*F*C*B*D*A*E*G*F*C*G*D* | A and A*D*B*F*C*A*D*G*E*B*F*E*G*C* | A.

But even this simplest amphicheiral form has other applications.

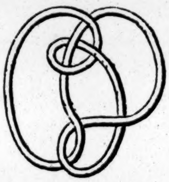
Thus, it will easily be seen that the figures below are mere *distortions* of fig. 2 above; and that, the dotted parts being fixed, they can be changed on an actual cord into one another, and even reversed (as from left to right), thus giving four distinguishable forms, of which I figure only two,—

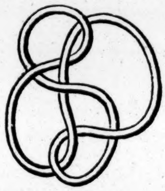




Each of these may be changed without undoing the fixtures to its reverse (from left to right) by the first simple process just described, the loop, in fact, being transferred from one branch of the (undotted) cord to the other.

Of course the number of ways in which the dotted part can be varied is infinite. I therefore give here only that which reproduces the forms already quoted from Listing as equivalent.





A glance shows that their type-symbols are

$$\left.\begin{array}{l} r^5+3r^3 \\ l^4+2l^3+2l^2 \end{array}\right\} \text{ and } \left\{\begin{array}{l} 2r^4+2r^3 \\ l^4+2l^3+2l^2 \end{array}\right.$$

which are those already quoted from Listing.

In fact, looking at Listing's figures above, we see that in each there is a part of the curve, containing four crossings, exactly the same as one or other of the two (partly dotted) distortions of the 4-crossing knot above.

The paper contains many other instances of these applications of amphicheiral forms.

In conclusion, it appears that the problem of finding all the absolutely distinct forms of knots, with a given number of intersections, is a much more difficult one than I at first thought; and it is so because the number of really distinct species of each order is very much *less* than I was prepared to find it. The question now belongs more to quantitative than to qualitative relations. It resembles, in fact, the species of problem originally suggested by Crum Brown, and resolved by Sylvester and Cayley, of determining the number of conceivable Hydrocarbons under given conditions of limitation. And here I am glad to leave it, for at this stage it is entirely out of my usual sphere of work, and it has already occupied too much of my time.

[*April* 11*th*.—Prof. Listing, to whom I sent in "proof" the above abstract of part of his paper, has kindly written some remarks upon it, from which I extract the following very interesting passages, which increase our regret that such a master has published so little on a subject which he has evidently made his own:—

"In dem . . . proof . . . sollte die Aufmerksamkeit vorzugsweise auf den allerdings sehr kurzen Theil gelenkt werden, welcher sich auf die Knoten oder Curvenverschlingungen bezieht. Anderenfalls würde ich gewünscht haben, dass das—obwohl sehr elementäre—Capitel über die 'Position' etwas näher besprochen worden wäre, zumal darin das Motiv zu finden, warum ich unsere gewöhnliche Schraube laeotrop nenne statt dexiotrop, wie Sie p. 307 rügen. Ich habe längst die Benennungen rechts und links bei Schraubenwindungen, sowie die lateinischen oder griechischen Ausdrücke, welche stets zu den widerwärtigsten Verwechselungen Anlass geben, durch die Namen *Delta*- und *Lambda*-Windung ersetzt, welche, wie p. 42 meiner Schrift erwähnt ist, mnemonish und intuitiv die Vorstellungen fixiren. Der Botaniker—und einige haben in der That diesen Modus befolgt—würde also dem Humulus lupulus Deltawindung, dem Convolvulus Lambdawindung zuschreiben, so wie unsere künstliche Schrauben der Technik meistens Lambda-Schrauben und nur in seltneren Fällen Delta-Schrauben sind. Die Unterscheidung zwischen Inversion und Perversion im Anschluss an sogenannte positive und negative Positionen ist nur ein gelegentliches specielles Ergebniss."

"Die Verschlingungen cyclischer Curven im Raume d. h. die Knoten betreffend, so sollten, wie in den 'Vorstudien' es nicht anders erwartet werden durfte, die Geometer auf die Bedeutsamkeit dieses überaus schwierigen Theils der Topologie durch Aufzeigung einiger der einfachsten Anfänge aufmerksam gemacht

werden, z. B. durch Andeutungen über mögliche Bezeichnungsmethoden, Symbole und dgl. durch welche wir, wie es die Wissenschaft in allen analogen Fällen zu erstreben hat, von der Intuition zu den Begriffen fortzuschreiten vermögen. Die von mir empfohlenen Symbole sollen nichts weiter sein als ein derartiger Fingerzeig, und wenn ich auch ganz mit Ihnen darin übereinstimme dass das Schema einen Nodalcomplex leichter construirbar macht als das Symbol, so bleibt doch das Schema viel weiter von dem Begriffe entfernt als das Symbol, auch abgesehen von den auf beiden Seiten noch übrig bleibenden Vieldeutigkeiten. Wie fragmentär das, was ich vor 30 Jahren darüber angedeutet habe, gewesen, kann ich Ihnen unter vielen an dem einen Punkte erläutern, dass ich mich dort nur auf die Verknotung einer einzigen cyklischen oder einer cyklodischen Curve beschränkte, und nur auf solche Verschlingungen unter der Benennung 'reducirt' einging, deren sämmtliche Parcellen (incl. Amplexum) monotyp sind, wiewohl Ihnen selbst bekannt ist, dass es Knoten giebt mit Maschen, deren Ecken promiscue δ und λ heissen, ohne auf einfachere Formen, d. h. eine geringere Zahl von crossings reducirbar zu sein. . . . Es versteht sich von selbst, das die in den Vorstudien angeführten Symbole nur auf monotype reducirte Complexe anwendbar sind, und durch Verallgemeinerung andere Gestalten annehmen, worauf ich indessen hier weiter einzugehen unterlasse."

I have thought it better to give Listing's own comments on my remarks than to seek permission from the Council to alter these remarks in the text.]